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Spectral functions of the disordered Tomonaga–Luttinger model

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Abstract. Using the functional integral bosonization technique, the spinless Tomonaga–Luttinger model with impurity forward scattering is solved exactly. Explicit analytical results are given for the one- and two-particle spectral functions in terms of a convolution integral representation. Sharp structures in the frequency spectra of an ordered system are smoothed out by the disorder. The existence of a charge–density instability is predicted if the critical exponent γ_{CDW} is less than -0.5. There is no influence of the disorder on the superconducting spectral function χ_{SC}.

1. Introduction

The optical and transport properties of quasi-one-dimensional (1D) systems, which are realized in organic conductors and quantum wires with an effective geometrical dimensionality of one, have recently occupied a great deal of attention. A pure 1D interacting electronic system is necessarily described by the Tomonaga–Luttinger (TL) model [1, 2], which exhibits non-Fermi-liquid behaviour so that its Green functions do not have quasi-particle poles but branch cuts instead. Applying the elegant boson representation of the fermion field operators, exact results were derived for given sets of coupling constants. Details of the large volume of work in this field can be found in the review articles [3, 4]. Recently, the exactly calculated Green functions of the TL model have been used to investigate the transport properties [5] and the Raman scattering [6] for 1D interacting electron systems. Comparing these theoretical results with experiments one has to realize that up to now it has not been clear whether finite-size effects in actual semiconductor quantum wires mask the low-energy physics of the TL model and restore the Fermi liquid behaviour of the electron gas.

It is expected that below a given crossover temperature impurity effects play an important role and give rise to 1D Anderson localization [5]. From a theoretical point of view it is an interesting task to study the interplay between electron–electron interactions and disorder because the interaction turns out to be relevant for the nature of the metal–insulator phase transition [7]. It has been stressed that near the transition neither the disorder nor the interaction between electrons is small so that simple models developed for disordered non-interacting electrons are not sufficient to understand all experimental data [7]. Results obtained for the disordered TL model could therefore shed some light on the difficult problem to consider disorder and interaction simultaneously. Giamarchi and Schulz [8] studied the interplay between disorder and electron–electron interaction on the basis of a perturbative renormalization-group (RG) approach by treating both impurity mediated forward and

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backward scattering. Whereas the electron-impurity forward scattering turns out to be of secondary importance, the backward scattering due to the random potential plays a crucial role in the localization process. If the electron–electron interaction strength is sufficiently attractive, a localization–delocalization transition is predicted [9, 8]. Theoretical results have been derived from the first-order RG flow equations [8]. The adequate description of the localized region requires, however, the consideration of higher-order renormalization equations, too.

The main objective of the present paper is to show that the functional integral bosonization technique worked out in [10, 11] provides an elegant mathematical tool to investigate disorder effects in 1D interacting systems. To show this we put emphasis on exact results, which can be derived by neglecting the impurity mediated backward scattering. Thus we focus our attention on the mathematical structure of the approach by disregarding the interesting physical processes associated with impurity backward scattering. The functional-integral bosonization method has also been applied successfully to a treatment of electron–phonon forward scattering in the spinless TL model [11, 12]. A second aim of our paper is to present new analytical and numerical results for the one- and two-particle spectral functions.

In the TL model there are two branches of fermion fields $\psi_1(x)$ ($\psi_2(x)$) describing electrons moving in the positive (negative) x direction with velocity v_F so that the free-electron dispersion relation is linearized around the Fermi energy. The Hamiltonian is given by

$$H = \int_0^L \mathrm{d}x\ \bar{\psi}(x)\left[-v_F(\mathrm{i}\sigma_3\partial_x + p_F) + u(x)\right]\psi(x) + \int_0^L \mathrm{d}x\,\mathrm{d}y \sum_{i,j=1,2} \bar{\psi}_i(x)\psi_i(x)V_{ij}(x-y)\bar{\psi}_j(y)\psi_j(y) \tag{1}$$

where $\psi(x)$ and $\bar{\psi}(x)$ are independent two-component Grassmann variables and V_{ij} ($i, j = 1, 2$) are the elements of the coupling matrix

$$\widehat{V} = \begin{pmatrix} V_1 & V_2 \\ V_2 & V_1 \end{pmatrix}. \tag{2}$$

Backscattering and umklapp processes with large momentum transfer have been neglected. The random potential $u(x)$ is Gaussian distributed with a white noise pair correlation function

$$\langle u(x)u(x')\rangle_u = u_0\delta(x-x') \tag{3}$$

where $\langle\cdots\rangle_u$ means the average over the Gaussian random potential. Exact results for the one- and two-particle spectral functions of this disordered spinless TL model are derived and discussed in sections 2 and 3.

2. One-particle spectral function

The imaginary-time action of the Hamiltonian (1) is given by

$$S\left[\bar{\psi}, \psi\right] = \int_0^\beta \mathrm{d}\tau \int_0^L \mathrm{d}x\ \bar{\psi}(x\tau)\partial_\tau\psi(x\tau) + \int_0^\beta \mathrm{d}\tau\ H(\tau) \tag{4}$$

where $H(\tau)$ is the Hamiltonian in the Heisenberg picture and $\beta = 1/k_{\mathrm{B}}T$. The one-particle Green's function of the disordered system is expressed by a functional integral over Grassmann fields

$$\mathcal{G}_i(x\tau, x'\tau') = \left\langle \int \mathrm{D}\bar{\psi}\,\mathrm{D}\psi\;\bar{\psi}_i(x\tau)\psi_i(x'\tau')\mathrm{e}^{-S[\bar{\psi},\psi]} \left[\int \mathrm{D}\bar{\psi}\,\mathrm{D}\psi\;\mathrm{e}^{-S[\bar{\psi},\psi]}\right]^{-1}\right\rangle_u . \tag{5}$$

Using the Hubbard–Stratanovich transformation the four-fermion term in (5) is expressed by a functional integral over real commuting fields ϕ_i so that the fermionic part can be integrated out, which provides, in both the numerator and denominator of the expression in (5), a product of Matthews–Salam determinants. For the TL model these determinants can be calculated exactly [10] and the Green's function may be represented in terms of a functional integral over Bose fields

$$\mathcal{G}_i(x\tau) = \langle G_i(x\tau, 00; [u + \mathrm{i}\phi])\rangle_{u,\phi} . \tag{6}$$

The average over the ϕ field is defined by

$$\langle\cdots\rangle_\phi = \int \prod_{i=1,2} \mathrm{D}\phi_i \ldots \exp(-S_{\mathrm{eff}}[u, \phi_i]) \left[\int \prod_{i=1,2} \mathrm{D}\phi_i \exp(-S_{\mathrm{eff}}[u, \phi_i])\right]^{-1} \tag{7}$$

where the following effective action had been introduced:

$$\begin{aligned} S_{\mathrm{eff}}[u, \phi_i] = &-\frac{1}{\beta L}\sum_{p\omega_n}\sum_i a_i(p, \omega_n)\left[\mathrm{i}\beta\delta_{n0}u(p)(\phi_i(p, \omega_n) + \phi_i(-p, -\omega_n))\right] \\ &+\frac{1}{\beta L}\sum_{p\omega_n}(\phi_1\,\phi_2)_{-p,-\omega_n}\Omega_0(p, \omega_n)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_{p\omega_n} . \end{aligned} \tag{8}$$

$u(p)$ is the Fourier transform of the random potential $u(x)$ and the matrix Ω_0 is given by

$$\Omega_0(p, \omega_n) = \frac{1}{V_1^2 - V_2^2}\begin{pmatrix} a_1(V_1^2 - V_2^2) + V_1 & -V_2 \\ -V_2 & a_2(V_1^2 - V_2^2) + V_1\end{pmatrix} \tag{9}$$

with

$$a_i(p, \omega_n) = \mp\frac{1}{4\pi}\frac{p}{\mathrm{i}\omega_n \mp v_{\mathrm{F}}p} . \tag{10}$$

Furthermore, the field-dependent Green's function G_i in (6) can be written as [10]

$$G_i(x\tau, 00; [u + \mathrm{i}\phi]) = G_i^{(0)}(x\tau)\exp\left[\frac{1}{\beta L}\sum_{p\omega_n}\chi_i(p, \omega_n)J_i(-p, -\omega_n|x\tau; 00)\right] \tag{11}$$

where

$$J_i(-p, -\omega_n|x\tau; 00) = \frac{1}{\mathrm{i}\omega_n \mp v_{\mathrm{F}}p}\left(\mathrm{e}^{\mathrm{i}(px - \omega_n\tau)} - 1\right) \tag{12}$$

and

$$\chi_i(p, \omega_n) = \beta\delta_{n,0}u(p) + \mathrm{i}\phi_i(p, \omega_n) . \tag{13}$$

$G_i^{(0)}(x\tau)$ is the Green's function of free electrons without any disorder ($u = 0$). The average over the random potential u is complicated by the fact that it appears in both the nominator and denominator in (6) via the average over the ϕ field. It is therefore expedient to introduce additional artificial degrees of freedom, which are removed at the end of the calculation but which allow us to handle the random field dependence resulting from the

denominator. Applying this so-called replica trick the configuration average can be carried through exactly. The replica representation of (6) has the following form:

$$\mathcal{G}_1(x\tau) = G_1^{(0)}(x\tau)\lim_{N\to 0}\int\prod_{l=1}^{N}\mathrm{D}\varphi^l\left\langle\exp\left(\frac{1}{L}\sum_p u(p)F(-p)\right)\right\rangle_u$$
$$\times\exp\left\{-\frac{1}{\beta L}\sum_{p\omega_n}\sum_{l=1}^{N}\varphi^{lT}(-p,-\omega_n)\Omega_0(p,\omega_n)\varphi^l(p,\omega_n)\right.$$
$$\left.+\frac{\mathrm{i}}{\beta L}\sum_{p\omega_n}\phi_1(p,\omega_n)J_1(-p,-\omega_n|x\tau;00)\right\} \tag{14}$$

where

$$F(-p) = J_1(-p,0|x\tau;00) - 2\mathrm{i}\sum_l\left(a_1(p,0)\varphi_1^l(p,0) + a_2(p,0)\varphi_2^l(p,0)\right). \tag{15}$$

Here $\varphi^{lT} = (\phi_1^l, \phi_2^l)(l = 1, \ldots N)$ are real two-component fields. Using (equation (3)) the Gaussian average over the random potential u is easily performed

$$\left\langle\exp\left(\frac{1}{L}\sum_p u(p)F(-p)\right)\right\rangle_u = \exp\left[\frac{u_0}{2}\frac{1}{L}\sum_p F(p)F(-p)\right] \tag{16}$$

and one arrives at a representation of the Green's function (6) in terms of a quadratic form in the Bose fields φ^l. Consequently, the functional integral over these fields is straightforwardly calculated and the one-particle Green's function is expressed by

$$\mathcal{G}_1(x\tau) = G_1^{(0)}(x\tau)\exp\left(-\frac{u_0}{2v_\mathrm{F}^2}|x|\right)$$
$$\times\lim_{N\to 0}\exp\left[\frac{1}{4\beta L}\sum_{p\omega_n} j^T(-p,-\omega_n)\Omega^{-1}(p,\omega_n)j(p,\omega_n)\right] \tag{17}$$

where j is a vector with $2N$ components

$$j^l(-p,-\omega_n) = iJ_1(-p,-\omega_n|x\tau;00)\left[\delta_{l,1}\begin{pmatrix}1\\0\end{pmatrix} - \frac{u_0\beta\delta_{n,0}}{2\pi v_\mathrm{F}}\begin{pmatrix}1\\1\end{pmatrix}\right]. \tag{18}$$

Ω is a special $2N\times 2N$ block Toeplitz matrix, its diagonal and off-diagonal parts are given by $\Omega_0+\Omega_1$ and Ω_1, respectively, where Ω_1 is the following 2×2 matrix

$$\Omega_1 = \beta\delta_{n,0}\frac{u_0}{8\pi^2 v_\mathrm{F}^2}\begin{pmatrix}1&1\\1&1\end{pmatrix}. \tag{19}$$

From the calculated inverse matrix Ω^{-1} the replica limit is easily read off and it remains only elementary but tedious to integrate over p and ω. Finally, a Wick rotation gives the following exact analytic form of the zero-temperature one-particle Green's function:

$$\mathcal{G}_1(xt) = \frac{\mathrm{e}^{\mathrm{i}p_\mathrm{F}x-\alpha|x|}}{2\pi\,\mathrm{i}}\left\{\Theta(t)\frac{x-v_\mathrm{F}t+\mathrm{i}\Lambda}{(x-v_\mathrm{F}t+\mathrm{i}\varepsilon)(x-vt+\mathrm{i}\Lambda)}\right.$$
$$\left.\times\left[\frac{\Lambda^2}{(x-vt+\mathrm{i}\Lambda)(x+vt-\mathrm{i}\Lambda)}\right]^{\gamma} + \Theta(-t)\ [\mathrm{CC}]\right\} \tag{20}$$

with the renormalized Fermi velocity

$$v = \sqrt{\left(v_\mathrm{F}+\frac{V_1}{4\pi}\right)^2 - \left(\frac{V_2}{4\pi}\right)^2} \tag{21}$$

and the exponents

$$\alpha = \frac{u_0}{2v_F^2}\left(1 + \frac{1}{1 + 4\pi v_F/(V_1 - V_2)}\right)^2 \tag{22}$$

$$\gamma = \frac{1}{2}\left(\left(v_F + \frac{V_1}{4\pi}\right)/v - 1\right). \tag{23}$$

As usual a momentum cut-off parameter Λ has been introduced, which sets the scale for the crossover from free-fermion behaviour to the Luttinger liquid. Employing the frequently used approximation $v_F \to v$ in (20) this expression simplifies further:

$$\mathcal{G}_1(xt) = \frac{e^{i p_F x - \alpha|x|}}{2\pi\,i}\left(\frac{\Theta(t)}{x - vt + i\varepsilon}\left[\frac{\Lambda^2}{(x - vt + i\Lambda)(x + vt - i\Lambda)}\right]^\gamma + \Theta(-t)\,\mathrm{CC}\right). \tag{24}$$

A shortcoming of this representation, which agrees with known results [13] for an ordered system ($\alpha = 0$), is an inadequate description of the high-energy properties of the model [14]. However, it is a suitable form to characterize the behaviour at low frequencies, in which we are mainly interested in. The random potential gives rise to an exponential decay of the correlation functions described by the characteristic exponent α, which also depends also on the parameters $V_{1,2}$ of the Coulomb interaction.

Analytical results for the spectral function are obtained by a double Fourier transformation of the Green's function (24). Introducing new variables ($s = x + vt$ and $s' = x - vt$) the integrals are easily calculated (see e.g. [14]) and the spectral function $A_1(q, \omega)$ may be expressed by the following convolution integral:

$$A_1(q, \omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} dq' \frac{\alpha}{\alpha^2 + (q' - (q - p_F))^2}\, A_1^{(0)}(q', \omega) \tag{25}$$

where $A_1^{(0)}(q, \omega)$ is the spectral function of the ordered system with $u = 0$:

$$A_1^{(0)}(q, \omega) = \frac{\Lambda}{v\Gamma(\gamma)^2} e^{-\Lambda|\omega - vq|/2v}\left(\frac{\Lambda}{2v}|\omega - vq|\right)^{\gamma - 1}\gamma\left(\gamma, \frac{\Lambda}{2v}|\omega + vq|\right)\Theta(\omega^2 - v^2q^2)\,. \tag{26}$$

Taking into account the properties of the incomplete gamma function $\gamma(a, x)$ it is seen that both A_1 and $A_1^{(0)}$ satisfy the sum rule

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A_1(q, \omega) = 1\,. \tag{27}$$

As according to (23) the exponent γ is always positive or zero the algebraic singularity in $A_1^{(0)}$ at $\omega = vq$ is smoothed out by the random potential. This suppression of divergencies is a characteristic effect of forward scattering by impurities, which could be modified by electron–electron umklapp scattering in a non-trivial manner [8]. An example of this suppression is shown in figure 1, where the spectral function of the ordered system $A_1^{(0)}$ (thin curve) is compared with $A_1(q, \omega)$ calculated from (25) (thick full curve) for $\gamma = 0.5$ and the dimensionless disorder parameter $\alpha\Lambda = 0.2$. The main disorder effect is to weaken the steep frequency dependence of the spectral function at the edges $\omega = \pm vq$. Both spectra do not exhibit any quasi-particle-like δ-peak structure, which could be identified by a quasi-particle as in the Fermi liquid theory.

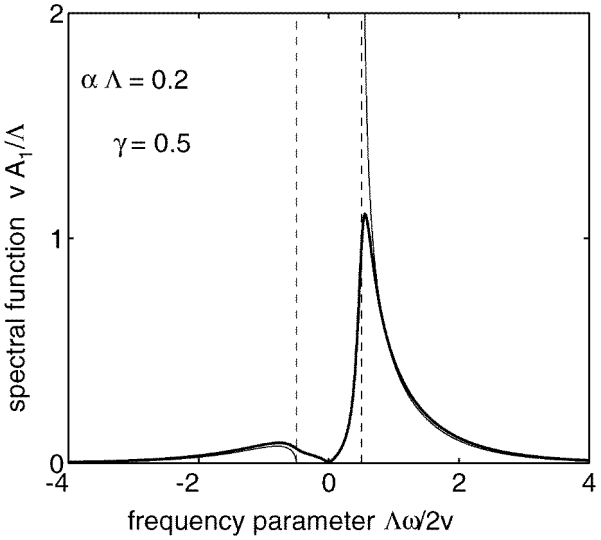

Figure 1. Frequency dependence of the one-particle spectral function (25) for the dimensionless disorder parameter $\alpha\Lambda = 0.2$ and $\gamma = 0.5$ (thick curve) compared with the spectral function $A_1^{(0)}$ (equation (26)) of an ordered system (thin curve). In addition it is $\Lambda(q - p_F) = 1$.

3. Two-particle spectral function

A strictly one-dimensional system cannot have a phase transition at finite temperature to a long-range ordered charge–density wave (CDW) or superconducting (SC) state. However, divergences in the corresponding two-particle spectral functions are a sign that a coupled many-chain system may undergo a transition to a CDW or SC state. Therefore, it is interesting to examine the zero-temperature susceptibilities of the system for any indication of instabilities. The CDW and SC two-particle spectral functions are defined by the following equations:

$$\chi^{>}_{\mathrm{CDW}}(x\tau) = \langle T_\tau \psi_1(x\tau)\bar{\psi}_2(x\tau)\psi_2(00)\bar{\psi}_1(00)\rangle \tag{28}$$

$$\chi^{>}_{\mathrm{SC}}(x\tau) = \langle T_\tau \psi_2(x\tau)\psi_1(x\tau)\bar{\psi}_1(00)\bar{\psi}_2(00)\rangle\,. \tag{29}$$

First the CDW susceptibility is considered. Following the same steps as in section 2 it is straightforward to derive the replica representation of the two-particle spectral function. As in section 2 the configuration average is carried out and one arrives at

$$\chi^{>}_{\mathrm{CDW}}(x\tau) = G_1^{(0)}(x\tau)G_2^{(0)}(-x-\tau)\lim_{N\to 0}\int D\varphi \exp\left\{-\frac{1}{\beta L}\sum_{p\omega_n}\varphi^T(-p,-\omega_n)\right.$$
$$\left.\times\Omega(p,\omega_n)\varphi(p,\omega_n) + \frac{1}{\beta L}\sum_{p\omega_n} j^T(-p,-\omega_n)\varphi(p,\omega_n)\right\} \tag{30}$$

where contrary to (18) the $2N$ components of the vector j are now given by

$$j^l(-p,-\omega_n) = \mathrm{i}\delta_{l,1}\left(J_1(-p,-\omega_n|x\tau;00)\begin{pmatrix}1\\0\end{pmatrix} + J_2(-p,-\omega_n|x\tau;00)\begin{pmatrix}0\\1\end{pmatrix}\right)$$
$$+\frac{\mathrm{i}u_0\beta\delta_{n,0}}{2\pi v_F^2}\frac{1}{p}\left(\mathrm{e}^{\mathrm{i}px} - \mathrm{e}^{-\mathrm{i}px}\right)\begin{pmatrix}1\\1\end{pmatrix}. \tag{31}$$

As the exponent in (30) is a quadratic form in the $2N$-dimensional vector φ the functional integral can be calculated exactly. The remaining p, ω integrals are elementary and after a Wick rotation the following compact expression for the CDW spectral function is obtained

$$\chi^{>}_{\mathrm{CDW}}(xt) = \frac{1}{(2\pi)^2}\frac{\mathrm{e}^{2\mathrm{i}p_{\mathrm{F}}x-2\alpha|x|}}{(x-vt+\mathrm{i}\varepsilon)(x+vt-\mathrm{i}\varepsilon)}\left[\frac{\Lambda^2}{(x-vt+\mathrm{i}\Lambda)(x+vt-\mathrm{i}\Lambda)}\right]^{\gamma_{\mathrm{CDW}}}. \quad (32)$$

Again there is only a disorder-mediated exponential decay of the spectral function [8] characterized by the exponent 2α. The critical exponent is determined by

$$\gamma_{\mathrm{CDW}} = \lambda_{\mathrm{CDW}} - 1 \qquad \lambda_{\mathrm{CDW}} = \sqrt{\frac{4\pi v_{\mathrm{F}} + V_1 - V_2}{4\pi v_{\mathrm{F}} + V_1 + V_2}} \quad (33)$$

and is independent of the disorder. The instabilities of the CDW susceptibility are considered in the Fourier representation, which also has the form of a convolution integral

$$\chi_{\mathrm{CDW}}(q,\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{d}q'\frac{2\alpha}{(2\alpha)^2+(q'-(q-2p_{\mathrm{F}}))^2}\,\chi^{(0)}_{\mathrm{CDW}}(q',\omega) \quad (34)$$

where $\chi^{(0)}_{\mathrm{CDW}}$ is the spectral function of an ordered system ($u = 0$)

$$\chi^{(0)}_{\mathrm{CDW}}(q,\omega) = \frac{\mathrm{sgn}(\omega)\mathrm{e}^{-\Lambda|\omega-vq|/2v}}{2v\Gamma(\gamma_{\mathrm{CDW}})^2}\gamma\left(\gamma_{\mathrm{CDW}},\frac{\Lambda}{2v}|\omega-vq|\right) \times\gamma\left(\gamma_{\mathrm{CDW}},\frac{\Lambda}{2v}|\omega+vq|\right)\Theta(\omega^2-v^2q^2)\,. \quad (35)$$

In the case of small arguments ($\Lambda|\omega \pm vq|/2v \ll 1$) the incomplete gamma function may be expended so that we obtain

$$\chi_{\mathrm{CDW}}(q,\omega) \cong \frac{\mathrm{sgn}(\omega)\alpha\Lambda\mathrm{e}^{-\Lambda|\omega|/v}}{2\pi v\Gamma(\gamma_{\mathrm{CDW}}+1)^2}\left(\frac{\Lambda\omega}{2v}\right)^{2\gamma_{\mathrm{CDW}}+1}\int_{-1}^{1}\mathrm{d}z\frac{(1-z^2)^{\gamma_{\mathrm{CDW}}}\mathrm{e}^{-\Lambda\omega(1-z)/2v}}{(\alpha\Lambda)^2+\left(\frac{\Lambda\omega}{2v}z-\frac{\Lambda}{2}(q-2p_{\mathrm{F}})\right)^2}. \quad (36)$$

Instabilities of the system are indicated by a divergence in the static susceptibility. Due to the effect of the random potential a divergence, which may appear in the susceptibility $\chi^{(0)}_{\mathrm{CDW}}$ of an ordered reference system and which thus indicates the formation of a CDW state, is completely smoothed out. Only if the exponent γ_{CDW} satisfies the condition $\gamma_{\mathrm{CDW}} < -0.5$ does the static CDW susceptibility diverge independent of the disorder strength. This behaviour is illustrated in figure 2 where the CDW spectral function is shown as a function of the dimensionless frequency parameter $\Lambda\omega/2v$ for the disorder strength $\alpha\Lambda = 0.05$ and the wave vector $\Lambda(q-2p_{\mathrm{F}}) = 1$. As long as $\gamma_{\mathrm{CDW}} > -0.5$ the spectral function is finite for all frequencies. Only if $\gamma_{\mathrm{CDW}} < -0.5$ (an example is shown in the inset of figure 2) does the spectral function diverge at $\omega = 0$ for all disorder strengths. This transition is driven by a Coulomb interaction quite similar to the Mott–Hubbard transition. Localization in disordered systems results, however, from impurity backward scattering, which we neglected in the present paper.

Contrary to the disorder influence on the charge–density wave state there is no effect of the impurity forward scattering on the SC susceptibility. The superconducting two-particle spectral functions of an ordered system and a system with disorder scattering in the forward

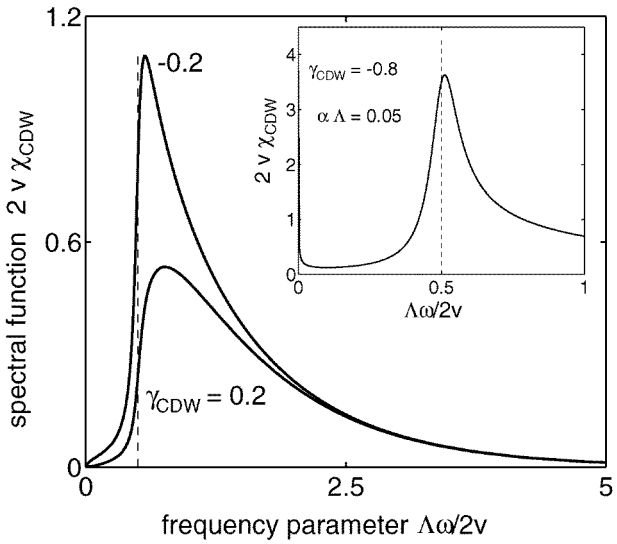

Figure 2. CDW spectral function (33) for $\alpha\Lambda = 0.05$ and $\Lambda(q - 2p_F) = 1$ and with $\gamma_{CDW} = 0.2$, -0.2 and -0.8 (inset), respectively. The band edge of the ordered system is marked by the broken line.

direction completely agree and are given by

$$\chi_{SC}(q,\omega) = \frac{\mathrm{sgn}(\omega)\mathrm{e}^{-\Lambda|\omega - vq|/2v}}{2v\Gamma(\gamma_{SC})^2}\gamma\left(\gamma_{SC}, \frac{\Lambda}{2v}|\omega - vq|\right)\gamma\left(\gamma_{SC}, \frac{\Lambda}{2v}|\omega + vq|\right)\Theta(\omega^2 - v^2q^2) \tag{37}$$

where the SC exponent is simply $\gamma_{SC} = 1/\lambda_{CDW} - 1$. This observation that the pairing fluctuations are not affected by the impurity forward scattering has already been stressed by Giamarchi and Schulz [8] and is in line with the fact that there is only a very weak disorder influence on the BCS superconductivity.

4. Summary

The method of functional integral bosonization in connection with the replica trick have been used to derive exact results for the spinless Tomonaga–Luttinger model with impurity forward scattering. It has been demonstrated that the functional integration provides an elegant mathematical tool to treat disorder effects in 1D systems so that it should be promising to apply this technique to physically more interesting models, which include backward scattering due to both electron–electron and electron-impurity scattering. The formalism has been used to derive analytical and numerical results for the one- and two-particle spectral functions. It has been shown that the effects of the impurity forward scattering on the spectral functions are easily represented by a convolution integral expression. Sharp structures and singularities in the frequency spectra are smoothed out by the random potential. Whereas there is a direct influence of the impurity forward scattering on the formation of charge–density wave states the superconducting spectral function remains unaffected by the random potential. The disorder prevents the formation of a CDW instability unless the exponent γ_{CDW} becomes smaller than -0.5. If $\gamma_{CDW} < -0.5$ the static susceptibility diverges indicating the existence of an instability.

We focused our attention on the case of small momentum transfer and neglected the backward scattering and umklapp processes. However, just the neglected terms mainly determine the physically interesting interplay between disorder and electron–electron interaction. We hope that the presented functional integral bosonization technique in connection with the replica trick can be applied successfully to an approximate treatment of more complicated disordered 1D Fermi systems with backward scattering.

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